Name: _____

SM3 6.4: Compose Functions

Problems: Given $f(x) = 2x + 5$ and $g(x) = 3x - 2$, simplify the expressions:

1) $(f \circ g)(x)$

$f(g(x))$
$2(3x - 2) + 5$
$6x - 4 + 5$
$6x + 1$

2) $(g \circ f)(x)$

$f(g(x))$
$3(2x + 5) - 2$
$6x + 15 - 2$
$6x + 13$

3) $(f \circ g)(2)$

$6(2) + 1$

13

4) $(g \circ f)(0)$

$6(0) + 13$

13

5) $(f \circ f)(x)$

$f(f(x))$
$2(2x + 5) + 5$
$4x + 10 + 5$
$4x + 15$

6) $(g \circ g)(x)$

$f(g(x))$
$3(3x - 2) - 2$
$9x - 6 - 2$
$9x - 8$

7) $(f \circ f)(9)$

$4(9) + 15$

51

8) $(g \circ g)(-1)$

$9(-1) - 8$

-17

Given $f(x) = \{(1,1), (4,2), (2,3), (3,4), (5,0)\}$ and $g(x) = \{(0,1), (1,2), (2,3), (3,4), (4,5)\}$, simplify the expressions.

9) $(g \circ f)(x)$

$f(x) = \{(1,1), (4,2), (2,3), (3,4), (5,0)\}$

$g(x) = \{(0,1), (1,2), (2,3), (3,4), (4,5)\}$

$\{(1,2), (4,3), (2,4), (3,5), (5,1)\}$

10) $(f \circ g)(x)$

$g(x) = \{(0,1), (1,2), (2,3), (3,4), (4,5)\}$

$f(x) = \{(1,1), (4,2), (2,3), (3,4), (5,0)\}$

$\{(0,1), (1,3), (2,4), (3,2), (4,0)\}$

11) $(g \circ f)(4)$

$\{(1,2), (4,3), (2,4), (3,5), (5,1)\}$

3

12) $(f \circ g)(2)$

$\{(0,1), (1,3), (2,4), (3,2), (4,0)\}$

4

Given $c(x) = \{(1,2),(2,3),(3,4),(4,1)\}$ and $d(x) = \{(1,3),(2,4),(3,1)\}$, simplify the expressions:

13) $(c \circ d)(x)$

$d(x) = \{(1,3),(2,4),(3,1)\}$

$c(x) = \{(1,2),(2,3),(3,4),(4,1)\}$

$\{(1,4),(2,1),(3,2)\}$

14) $(d \circ c)(x)$

$c(x) = \{(1,2),(2,3),(3,4),(4,1)\}$

$d(x) = \{(1,3),(2,4),(3,1)\}$

$\{(1,4),(2,1),(4,3)\}$

15) $(c \circ d)(2)$

$\{(1,4),(2,1),(3,2)\}$

1

16) $(d \circ c)(3)$

$\{(1,4),(2,1),(4,3)\}$?

$\emptyset$

Given $r(x) = 4x$, $s(x) = x^2$, and $t(x) = x - 3$, simplify the expressions:

17) $(r \circ s)(x)$

$r(s(x))$
$r(x^2)$
$4(x^2)$
$4x^2$

18) $(s \circ t)(x)$

$s(t(x))$
$s(x-3)$
$(x-3)^2$
$x^2 - 6x + 9$

19) $(r \circ s \circ t)(x)$

$r(s(t(x)))$
$4((x-3)^2)$
$4(x^2 - 6x + 9)$
$4x^2 - 24x + 36$

20) $(r \circ t \circ s)(x)$

$r(t(s(x)))$
$4((x^2) - 3)$
$4(x^2 - 3)$
$4x^2 - 12$

21) $(s \circ s \circ s)(x)$

$s(s(s(x)))$
$((x^2)^2)^2$
$(x^4)^2$
x^8

22) $(t \circ r \circ s \circ t)(4)$

$t\left(r(s(t(4)))\right)$
$\left(4((4-3)^2)\right) - 3$
$\left(4(1^2)\right) - 3$
$\left(4(1)\right) - 3$
$(4) - 3$
1

Express the function in the form $f \circ g$ by identifying what $g(x)$ and $f(x)$ are.

23) $F(x) = (x-9)^5$

$g(x) = x - 9$
$f(x) = x^5$

24) $F(x) = \sqrt{x} + 1$

$g(x) = \sqrt{x}$
$f(x) = x + 1$

25) $G(x) = \dfrac{x^2}{x^2+4}$

$g(x) = x^2$
$f(x) = \dfrac{x}{x+4}$

26) $G(x) = \dfrac{1}{x+3}$

$g(x) = x + 3$
$f(x) = \dfrac{1}{x}$

27) $H(x) = |1 - x^3|$

$g(x) = 1 - x^3$
$f(x) = |x|$

28) $H(x) = \sqrt{1+\sqrt{x}}$

$g(x) = 1 + \sqrt{x}$
$f(x) = \sqrt{x}$

Application: You are an investment broker at Wytiaz Brokerage Firm with access to information about how different banks will pay for access to your clients' funds. Goliath National Bank (GNB) pays $52 if you invest your money with them for a year. First Brooklyn Savings Bank (FBSB) pays 5% of your investment if you invest your money with them for a year.

Write a function, $g(x)$, that models the money you would have in the first bank (GNB) after one year:

29) $g(x) = x + 52$

Write a function, $f(x)$, that models the money you would have in the second bank (FBSB) after one year:

30) $f(x) = 1.05x$

Write a composition of functions that models the money you could earn by investing for a year in the second bank, then withdrawing your funds and investing in the first bank for a year:

31) $(g \circ f)(x) = g\big(f(x)\big) = g(1.05x) = 1.05x + 52$

32) An important client's daughter has $1000 to invest for two years. Mr. Wytiaz wants you to come up with the best plan for her yearly investments in order to maximize her money. Devise such a plan and write a sentence or two explaining why your plan is the best plan. Note that when writing to your boss, you should try and seem impressive and worthy of promotion!

Mr. Wytiaz,

Thanks for the opportunity to help out! I calculated all of the different ways we could manage the fund:

GNB for 2 years	$(g \circ g)(x) = (x + 52) + 52 = x + 104$	$(g \circ g)(1000) = 1000 + 104 = \$\mathbf{1104}$
FBSB for 2 years	$(f \circ f)(x) = 1.05(1.05x) = 1.1025x$	$(f \circ f)(1000) = 1.1025(1000) = \$\mathbf{1102.50}$
GNB then FBSB	$(f \circ g)(x) = 1.05(x + 52) = 1.05x + 54.6$	$(f \circ g)(1000) = 1.05(1000) + 54.6 = \$\mathbf{1104.60}$
FBSB then GNB	$(g \circ f)(x) = (1.05x) + 52 = 1.05x + 52$	$(g \circ f)(1000) = 1.05(1000) + 52 = \$\mathbf{1102}$

I confidently conclude that we should invest in GNB for one year, and then move the resulting funds to FBSB. I'd be glad to discuss the details of the fund with the clients personally and welcome them aboard. If you give me the green light I'll get the contact info from your secretary.

Thanks for your time.